



Forecasting Using the Markov Chain Transition Matrix for Exchange Rate Fluctuations

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Abstract. The current research is preparing an economic statistical study on exchange rate fluctuations for the period from (1996 to mid-2005) according to the well-known phrase: (The present is the past and the future is the present). It is a modest attempt to crystallize this type of price in a scientific-statistic method by adopting the matrix of transitional probabilities in Markov chains, as these chains are among the models that are relied upon in the process of forecasting when the data are in the present time and in three cases: - High - Low - Stability. Between the past, the present and the future, the researcher was able to reach in his research to the analysis by applying the method of the greatest places, and to achieve the main purpose of this research, the researcher dealt with four chapters as follows: The first chapter dealt with the problem of research, its importance, need, purpose and determination of its terms. The second chapter reviewed the previous studies published on the subject, as well as the theoretical aspect, in which the matrix of transitional and stable possibilities, the state of stability and independence of the Markov chains, and the estimation of transitional possibilities in the greatest possible way were presented. As for the third chapter, it included the practical aspect and the field application. The data related to the research procedures were collected from the date of (17/10/2003 to 30/6/2005), in which the Iraqi currency was exchanged from the old to the currency currently in circulation, and in which the situation of the Iraqi market suggested great relative stability at the time. Finally, the fourth chapter of this research included the conclusions reached by the researcher from which it came: The market was characterized by a state of stability after the rise in the exchange rate against the dollar, which confirmed the accuracy of the results of the matrix of transitional possibilities, which was reflected on the local stock exchange, as well as the development of recommendations that can be used by the relevant parties.

Keywords: Forecasting and Exchange Rate.

CHAPTER ONE: (RESEARCH METHODOLOGY)

First: Research Problem

The problem of the current research in the currency exchange market is determined by sudden fluctuations, which depend on the frequent news that comes through the various media or the instructions of the official authorities, which quickly affects the rise and fall in the exchange rate, which the researcher has touched through his dealings in the market in addition to the large gap between the official price of the state and the parallel price adopted in 2023, or the non-surprise associated with the flow of the usual market exchange rate, which was characterized by relative rise and decline according to supply and demand conditions.

Therefore, the problem of the current research becomes clear by trying to identify the exchange rate mechanism in the local stock exchange.

Second: The importance of research

The importance of the current research stems from the importance of the exchange rate traded in the local markets, all because the exchange rate is one of the most important economic indicators that exert a significant impact on the overall economic life, through which local goods and services are evaluated by investors and traders, and this price is of interest to every member of society, especially when there are severe and accelerating changes in it, as many of us take the question about it continuously, and we ask about time, through which assets are periodically equalized. The talk about the Iraqi dinar and the fluctuation of its exchange rate in the market is one of the most prominent concerns that dominate the daily conversations of Iraqis, so this market was formed at the beginning of 1992 and the work was secretly and secretly until the beginning of 1996 when the state

began to intervene directly and publicly by selling the dollar from Iraqi banks at a price lower than the local trading rate, which led to the collapse of the exchange rate to a very large extent, as the highest exchange rate at the beginning of 1996 amounted to 3000 thousand dinars per dollar and the majority of us remember these days and the economic effects they left. This collapse was called Ramadan because of its occurrence in the month of Ramadan and therefore the exchange rate fell to 500 dinars per dollar.

The years of the siege in Iraq have had a great impact on the fluctuation of the Iraqi dinar exchange rate printed inside the country without a cover in front of the foreign currency, the dollar, and its instability, as a result of the rise and fall in this price, and the significant changes that occurred to it, which directly affected the decline and decline in the well-being of the Iraqi citizen at the time.

Since the exchange rate of the Iraqi dinar was highly fluctuating, this process must be linked to a statistical pattern and by obtaining real data that was analyzed statistically using the probabilistic laws followed by accidents that occur frequently. Hence, the Markov series were of great importance for our research in practice. These series occupy an important place and are considered one of the distinct statistical methods, and the Markov probability model or Markov series, especially the matrix of transitional probability, and have become used as a base to describe the data set divided into cases for a time series.

The extrapolation of the market situation in the long term through the data used and that the entire random process falls within a purely statistical scope (probability, prediction, forecasting and guessing) and these topics are at the heart of the statistical process and the impact of statistics

emerged in providing the best advanced methods using Markov chains that gave real results, and that random processes and theories of these processes and applications and the issues they address are the result of human attention to the random aspect that falls outside his control, and his continuous and diligent attempt to reduce the impact of this aspect and make it as little as possible and predictions that he gets, and therefore the decisions that may be reached to address the problem.

The history of this market is recent for other markets, as the birth of the hour and the difficult economic situation that Iraq went through at the time, so this market has become a source of livelihood for many to this day by granting leave to practice the profession of banking, but the exchange market after the change of currency was characterized by stability regardless of the surprises, as a result of the central bank's control of the supply and demand process through daily banking auctions and has been called relative stability.

Third: The Need for Research

Through the results of the current research, which are related to the exchange rate of the Iraqi dinar against the dollar, the researcher believes that this type of research is needed by the concerned authorities such as (the Ministry of Finance, the Central Bank of Iraq and government and private banks), to review it and try to apply it to reach the possession of scientific and practical information through the results of its applied procedures in this field by building a matrix of transitional possibilities and the conditions that must be met in this matrix in order for the researcher to reach its scientific stability.

Fourth: Research Plan

The current research seeks to predict the exchange rates of the Iraqi dinar by studying the prices of the past and present using Markov series and applying the matrix of transitional probabilities in its three cases (rise, fall, stability) and estimating them in the manner of the greatest possible because it gives the appreciation that has the greatest probability.

TERMINOLOGY

Here, some of the terms that the researcher deems it important to identify from the relevant authorities, such as (the Ministry of Finance /the Central Bank of Iraq /government and private banks), will be sought as follows:

First : Exchange Rate

It means: (The mechanism followed globally and locally in the exchange of currencies is sufficient to follow the prevailing standards in the markets).

Or it is the percentage of exchanging a certain currency for another currency or currencies, which is just the price and currencies that have one exchange rate against the currencies of other countries, regardless of the different transactions, purposes and places.

There are two types: the first is the official price of the currency and the second is the free price of the currency.

Second : Forecasting

It means: (Forecasting and deducing scientifically by applying laws and probabilistic theories).

Third: Markov Chains [22,19,15,14,6] (Markov Chains)

Markov series: It is a plurality of a series (a series with a limited or unlimited number of cases , as time (T) is the parameter that distinguishes Markov chains, and the transition in one stage is defined as the case of Markov series with intermittent time $[X_n, n=0,1,2,---]$ that takes a specific countable number of

possible values represented by a set of non-negative integers, as the probability of (X_{n+1}) is in case (j), knowing that (X_n) in case (i) is symbolized by the symbol $P_{ij}(n,n+1)$.

$$\{ X_n=i / j = X_{n+1} \} p = P_{ij}(n,n+1) \quad (1)$$

CHAPTER TWO : (PREVIOUS STUDIES AND THEORETICAL ASPECT)

This chapter includes two main axes:

First: Previous Studies:

The theoretical part

First: Previous Studies

The initial research of the theory of probability in scientific research began in the late fifteenth century, specifically in 1494, as a result of the study of some different games of chance. At the beginning of the twentieth century, research in the theory of probability began to take on a logical philosophical dimension besides its mathematical dimension. This accelerated the progress of mathematical research in general towards the applied side. New concepts of probability were developed that are consistent with its statistical, not inductive, meaning. There was also a prominent role in determining the counting systems on which the theory of probability depends, as well as determining the conditions that must be achieved by scientists, such as talking about physical space and events.

Although the history of probability theory is relatively old, probability theory has become one of the branches of mathematics that is advancing at a rapid pace and its theoretical results have opened the door to the use of its various methods in practical applications that encounter a large number of workers in various fields.

Probability theory does not aim to predict the occurrence or non-occurrence of the only accident because it simply cannot do so, but it is known in practice when a large number of random accidents occur that it follows a probabilistic law.

In 1957, the researchers Anderson and Goodman [9] published a book in which they presented that the short time series data (Micro Data) can be estimated using the Greatest Possibility method and that the transition from state (i) to state (j) is known and provided ways to test the stability of the data in the partial time series (Micro Data).

In 1971, the researcher (Henry) [10] conducted a research on the Markov series with stable transition possibilities, as the researcher indicated that when taking into account the heterogeneous Markov series, there is an assumption, which often requires that he mediates between the steady state and the state of absolute change of the transition possibilities.

In 1972, Lee and Judge [11] presented research aimed at estimating the transition probabilities of Markov chains when the probabilities are unstable, in addition to determining efficient estimates of the transition probabilities of Markov chains when the available data are short time series data (Micro Data).

(N n,m) It is: -

$$P^m P^n = P^{(n+m)} \tag{5}$$

Whereas: -

$P^{(n+m)}$ represents the matrix of transitional probabilities of Markov strings after (m+n) of steps. The element in row (i) and column (j) of the matrix P^{m+n} is: -

$$P_{ij}^{(n+m)} = P_{ij}^{(n+m)} \tag{6}$$

per $0 < n, m$

The equation (6) is called the Chapman Kolommogrov Equation, and this equation [52] defines: -

$$P = 0 \leq j, i, 0 \leq n : P_{ij} \tag{7}$$

(Process in case i and case j after n transition probabilities) The Gapman-Colmcroff equation (k-c) is a method for calculating n of steps for transition probabilities P_{ij} .

Stationarity [3] Stationarity

Stochastic processes are distinguished from each other by specifications and assumptions that may make them useful for this or that applied aspect, and there are some applied aspects that can be described and addressed during more than one stochastic process, and this, of course, is a stop to what is intended by those applied aspects and the goal of the researcher.

So the concept of stationarity generally means that the statistical qualities of the stochastic process do not change to one degree or another over time.

Steady State] 7,6,5,2 [Steady State

The steady state appears when the stochastic process continues for a long time, as the ratio of the number of visits to each case stabilizes at a certain value, called the stable probabilities of that case, that is, the behavior of P_{ij} appears when $\infty \rightarrow n$ Therefore, the stable distribution is defined as follows :

The probabilistic vector $(u(s))$ ---2 $(u(s), 1(u(s) = u(s))$ which achieves $(u(s) = p^*(u(s))$ as a stable distribution of the stochastic process (Stationary Distribution).

When P is the matrix of transitional probabilities of Markov series with n finite states: -

$$\begin{matrix} U \\ u = u = p^{(m)} \text{ Lim} \\ M.. \\ U \end{matrix} \tag{8}$$

The only probabilistic vector is: -

$$1 > u_j > 0 \text{ and } 1 = u \text{ } \overline{1} \text{ (n u---2 u , 1 u) = u} \tag{9}$$

It represents the stable distribution of the process.

That is, if m approaches infinity, the probabilities of moving with m from the steps P_{ij} will depend on the last case and not the initial case, and this means that after a large number of attempts, the chain reaches a state of stability (Stationary).

Whereas: -

u The matrix of its rows is symmetrical and each row is the vector u , and the probabilistic vector u represents, as mentioned above, the stable distribution of the process .

Independent For Markov Matrix

The transitional probabilities during n of the steps can be obtained by multiplying the matrix P by itself n times and because of the independence of the variable X_{n+1} from the variables X_n, \dots, X_1, X_0 Hypothetically: From the definition of the Markov series, we find that $\{0 \leq n : X_n\}$ This series is represented by the transition matrix: -

$$\begin{matrix} P_2- P_1 \\ P_2 P_1 P_0 = P \\ P_2- P_1 \end{matrix} \tag{10}$$

Note that all rows of matrix P are identical , and that $P = P^m$ for all values of $1 \leq m$.

On the other hand, if $P_j = (j, i) P$ for all the values of (i, j) , that is, all the rows of the matrix (P) are identical, then it means that the discrete random variables (discontinuous) and from the definition of independence, are independent, which is a special case of the Markov series.

$$X_n, \dots, X_1, X_0 \tag{11}$$

It has a general distribution of } I } \overline{1} j, i P {

The Markov series is only a series of random variables, as for each N n, the future state X_{n+1} is independent of the previous cases

$$X_{n-1}, \dots, X_1, X_0 \tag{12}$$

provided that the current state X_n is known.

Estimating transition probabilities [18,5,4,3,2] Estimating of transition probabilities

Transitional probabilities are parameters that must be estimated. Markov's method is one of the appropriate models for time series data when the data are available for each time period of the series. These data represent cases in which the studied phenomenon falls and there is movement or transition of the phenomenon between the cases, for example, three cases in which we can estimate the transitional matrix. We will use a method of estimation, which is the Greatest Possibility Method (M.L.E), and it is one of the important methods in

statistical theory when estimating milestones because it gives us the estimate that has the greatest probability. Estimating by Maximum likelihood Method [18]

The transition of this type of data is from state (i) at time (t) to state (j) at time (t+1). We assume that there is a sample of observations in the form of Markov strings, and assuming that the number (0) ni represents the elements seen in the case (i) at the time (0 = t), as the observed elements indicate a series of cases at the time (T--2, 1, 0= t). Therefore, the Markov strings are in a stable state as follows: -

$$(X_{t-1}/X_t)Pr \text{ II}) X_0 (Pr =), X_T \text{---} Pr(X_0, X_1 \quad (13)$$

Let nij(t) represent the number of items viewed per (Xt=j, Xt-1 = i Wan: -

$$\text{---} (t) n_{ij} \text{ } \circ \text{I} \text{ } n_{ij} = \quad (14)$$

If nij(t) per (t,i,j) is known , we can obtain estimates of stable transition probabilities with the condition met

$$n \text{---} 1,2=j ; P_{ij} = 1 \text{ } \circ \text{I} \quad (15)$$

Since $1 = p_{ij}$ -

$$\lambda \text{ } i = n_{ij} \text{ } \circ \text{I} \quad (16)$$

By substituting Equation (16) into Equation (11), we obtain: -

$$p_{ij} = n_{ij} / \text{ } \circ \text{I} \text{ } n_{ij} \geq 0 \quad (17)$$

CHAPTER THREE -THE PRACTICAL ASPECT: (BUILDING THE TRANSITION PROBABILITY MATRIX)

Introduction

This study is a scientific attempt to cases of price movement in the currency exchange market related to research expressed by the data collected by direct follow-up of prices at the time, and according to the famous saying that : (the present is the reaping of the past and the future is the implantation of the present) The researcher worked to crystallize this saying in a smooth scientific method by adopting (Markov chains) in theory and practice, where between the past, the present and the future, and when applied, the researcher was able to reach the desired goal in analyzing the cases of rise, decline and stability of the exchange rate.

The months of the year were calculated on the basis of 365 days, and the year 2003 was divided into political events that took place in it and thus economic changes such as lifting the economic siege and changing the currency. In view of the political changes that took place, the period under study was divided into four stages and the transitional matrix for each stage was estimated in the manner of the greatest possible as follows :

1. The beginning of the year 1996 until 9/4/2003.
2. 10/4/2003 to 16/10/2003.
3. On 17/10/2003 until 30/6/2005.
4. Generally from the beginning of 1996 until 30/6/2005.

The third stage was relied on only for its novelty at the time for the immediate circumstances, and the study of Markov chains took the present only for the future.

As for the study of the previous two stages and the general stage, they are previous years of a related time series for the purpose of knowing the mechanism of work.

Transition Probability Matrix

The transition probability matrix is a reflection of the price movement during the years under consideration, as the basis of analysis in the Markov series is the formation of the matrix numbers first and then the estimation of the transition probability matrix second and we will code it with the symbol p and [p =] pij.

This matrix is a square of grade 3x3 due to the presence of three cases and is called the matrix of transitional possibilities ending for the Markov series stable in the following (18) : -

$$\begin{matrix} E3 & E2 & E1 \\ P13 & P12 & P11 \\ P23 & P22 & P21 \\ P33 & P32 & P31 \end{matrix} E = \quad (18)$$

Whereas: -

E1 = Price Rise Status.

E2 = Price drop condition.

E3 = Price Stability.

Definition of elements of the Marco F matrix

The elements of the transitional probabilistic matrix estimated by rows for each case are :

1. Elevation Status (E1)
 - P11 = The probability of the price rising after it was high.
 - P12 = The probability of the price falling after it was high.
 - P13 = The probability of the price stabilizing after it was high.
2. Decrease Status (E2)
 - P21 = The probability of the price rising after it was low.
 - P22 = The probability of the price falling after it was low.
 - P23 = The probability of the price stabilizing after it was low.
3. State of Stability(E3)
 - P31 = The probability of the price rising after it was stable.
 - P32 = The probability of the price falling after it was stable.
 - P33 = The probability of the price stabilizing after it was stable.

Matrix elements:

It is clear from the above that the transitional matrix can be found in (19) machines:

Height Decrease Stability
 Height Height Height Height Height Stability
 (19)--- Decrease Decrease Decrease Decrease Stability
 Stability Stability High Stability Low Stability Stability

Since the situation was abnormal in Iraq politically and economically and the changes that took place led to significant effects in the economic and monetary aspect for the Iraqi dinar, so the period was divided into three stages and only one of them was chosen for the requirements of deduction from the original research and its adoption in the following estimate:

M.L.E Estimation

In this method, we rely on the available short data (Micro Data) and we will use the formula (17) for estimation and the matrix (12) for stability for the stage that corresponds to the procedures of this research.

The matrix of transitional numbers for prices from today 17/10/2003 until today 30/6/2005 is in the numbers indicated and for the period (1 year + 8 months + 15 days) equivalent to 623 days as in (20) .

$$\begin{matrix}
 \text{Mug E3 E2 E1} \\
 125 \ 28 \ 45 \ 52 \ \text{E1} \\
 257 \ 29 \ 152 \ 46 \ \text{E2} = \text{P} \\
 241 \ 170 \ 37 \ 34 \ \text{E3}
 \end{matrix} \tag{20}$$

1.- J

From the matrix (20) we can estimate the matrix of transitional probabilities of prices, which is the Markov matrix for the period (1 year +8 months + 15 days) as in (21), which means the present.

$$\begin{matrix}
 \text{E3 E2 E1} \\
 0.22 \ 0.36 \ 0.42 \ \text{E1} \\
 (21) \text{---} \ 0.23 \ 0.59 \ 0.18 \ \text{E2} = \text{P} \\
 0.71 \ 0.15 \ 0.14 \ \text{E}
 \end{matrix}$$

We also obtained the stable matrix and the stability was in step No. (20) as in the attached appendix and the stable distribution vector of the process :

$$220.438 \ 0.348 \ 0.213 \ [= \text{Uj}$$

The matrix of transitional probabilities (21) above for its novelty at the time and when it reached the stable distribution vector (22) above and in the long term, and since the Markov chains depend on the present for the future for a certain period of time, the probability of stability is (0.438), the probability of decline is (0.348), and the probability of rise is (0.213), which shows that the probability of the state of stability is the highest and most likely, followed by the state of decline and finally the state of rise.

The probability values of the rise, fall and stability of the exchange rate in our research under study, represented by matrix No. (21) and the stable distribution vector No. (22), are what were adopted in the process of predicting the future exchange rate because they reached a state of conformity of the rows and columns of the matrix of transitional possibilities, as is clear and means independence when the random variables are separate (discontinuous), which is a special case of Markov chains.

The stable distribution vector No. (22) in its probability values, is the one that gave the correct scientific prediction that we aspire to in this study because it expressed a tangible and logical scientific result through field work in which data were collected and the use of laws and theories for the matrix of transitional probabilities.

CHAPTER IV : (CONCLUSIONS AND RECOMMENDATIONS)

This chapter includes a presentation of the conclusions reached by the researcher through the procedures of his research and the recommendations made to the

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relevant authorities (the Central Bank of Iraq and the markets that follow the speculation system) by adopting their results to identify the predictive possibilities and the following is a review of them :

CONCLUSIONS

Through the results of the researcher in this theoretical and field study of the currency exchange market in Baghdad, it was found that the transition probability matrix No. (21) and in the direction of stable distribution (U (Stationary Distribution) No. (22)). When the rows and columns matched, which is an important feature in the Markov series, the researcher was able to predict it for a specific period of time, which he specified by four years. The results were identical and accurate as mentioned, as the probability of the state of stability was the highest because it came in the first place and the probability of the state of decline, which came in the second place. As for the probability of the state of rise, it was in the third place.

The impact of the new currency was clear and effective in the values of transitional possibilities when it reached conformity after being hit and stabilized in step (20) and as shown in the accompanying appendix, which was reflected in the overall economic life in Iraq. It must be noted that the state of the Iraqi market, as everyone knows before the exchange of currency and at the time of the siege, was characterized by a state of rise only and a little decline, as it helped to absorb the very large quantities of the Iraqi dinar, which was floating in the market at the time because it was printed inside the country without a cover and its value was equal to the paper on which it was printed.

The researcher concludes from the phenomenon of the stability of the market situation that the Iraqi dinar has recovered and strengthened against the dollar and has become traded in international markets along with other currencies, which was touched by the Iraqi citizen from 2010 to 2020, where the official exchange rate stabilized at 1190 Iraqi dinars and the parallel price at 1200 Iraqi dinars and then in 2020 on the 20th of December, the official price was determined by the Central Bank of Iraq at the price of 1450 Iraqi dinars and the parallel at 1470 Iraqi dinars, and we also conclude that the stable distribution vector No. (22) proved its accuracy for the post-research years in order to stabilize the exchange rate for the above years.

It should be noted here that there was a sudden change in the third month of 2023 at the exchange rate, as the Central Bank of Iraq set the official price at 1320 Iraqi dinars in sending money abroad and buying goods by importers. When the citizen traveled abroad, the parallel price took different prices very high. Therefore, the state began to put an end to this by reducing it and gave a clear message to the power of the dinar against the dollar, and that direct dealing in the market is the Iraqi dinar only with strict control over speculators and the prevention of black transfers abroad and to reduce the large gap between the official price and the parallel price so that this price is close or equal.

Recommendations

1. The researcher recommends the relevant authorities (the Ministry of Finance and the Central Bank of Iraq) to adopt this study to know the predictive possibilities for the future, as we can predict the use of the transitional matrix of the Markov chains for the future and for a certain period of time specified by this study, even for four years.
2. It is also recommended to adopt this study in markets that are characterized by a speculative system and are usually official As in the Arab Gulf countries and the world.
3. It is recommended that they be used to predict future gold prices.
4. The researcher recommends their adoption by the state to establish private banks, which have already been established in Iraq, because they represent a successful start to investment in Iraq, which would increase the demand for the new Iraqi currency and that the flow of funds and investments in Iraq facilitates the entry of the Iraqi dinar as a currency traded in some Arab and foreign exchange markets.

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